Concepts of Mathematics (21-127)

[redacted]

Class info
Time: Weekdays at 9:00am–10:20am
Location: Hamerschlag Hall B131
Units: 10 units

Instructor info
Instructor: [redacted]
Office: [redacted]
Email: [redacted]

Website: [redacted]

1 Course content

Leaning objectives

Upon successful completion of this course you should be able to:

(1) Accurately use standard mathematical notation and terminology in mathematical writing, including symbolic logic, sets and set operations, binomial coefficients and factorials, and modular arithmetic;

(2) Identify feasible techniques for proving a proposition based on its logical structure;

(3) Write correct, clear and precise mathematical proofs, in an appropriate level of detail, of both familiar and unfamiliar results from pure mathematics in the areas covered;

(4) Evaluate proofs of mathematical statements in terms of the features that make them effective or ineffective;

(5) Recognise and apply standard proof techniques, including contradiction, contraposition, weak and strong mathematical induction, and the well-ordering principle;

(6) Accurately recall definitions and state and prove theorems in the mathematical areas covered;

(7) Solve standard unseen problems in the mathematical areas covered by identifying which results from the course are appropriate and applying them.

List of topics

- **Language of mathematics.** Logic: propositions, logical operators, truth tables, logical equivalence, logical formulas, quantifiers. Set theory: sets, set operations, number sets.
- **Natural numbers.** Induction and recursion: definition by recursion, proof by weak induction, proof by strong induction, well-ordering principle. Sizes of finite sets. Counting arguments: multiplication principle, addition principle, inclusion–exclusion principle. Binomial coefficients, factorials, permutations.
- **Integers.** Division: divisibility, division theorem, greatest common divisors, Euclidean algorithm, Bézout's lemma. Prime numbers: primes, irreducibles, fundamental theorem of arithmetic, infinitude of primes. Modular arithmetic: congruence, multiplicative inverses, coprime pairs, Fermat's little theorem.
- **Functions and relations.** Relations: graph of a relation, reflexivity, symmetry, transitivity, equivalence relations. Functions: graph of a function, composition, identity, image, injectivity, surjectivity, bijectivity. Cardinality: definition via injective functions, Cantor–Schröder–Bernstein theorem, Cantor's diagonalisation argument.
- *(Time permitting)* **Sets with structure.** High-level survey of partially ordered sets, groups, and/or metric spaces.
- **Proof techniques.** Logical structure of propositions, contradiction, contraposition, counterexamples, weak induction, strong induction, well-ordering principle, double-counting, double-containment.
- **LaTeX.** Text mode and math mode, mathematical symbols used in the course, sections, paragraphs, lists, theorem environments, labels, basic text formatting, aligned equations.

Supplementary reading materials

There are no required texts—all reading material you need will is contained in the course notes—but if you want to really excel then you'll probably want to do some supplementary reading of your own. The following books are particularly useful:

- *A Concise Introduction to Pure Mathematics* by Martin W. Liebeck;
- *How to Prove It: A Structured Approach* by Daniel J. Velleman;

A fun, non-technical discussion about how the skills you acquire when studying abstract mathematics relate to the real world is:

- *How Not to Be Wrong: The Power of Mathematical Thinking* by Jordan Ellenberg.

2 Assessment and grades

This course will be assessed by means of pre-class assignments, problem sheets, classwork and a final project. It will feel like a lot of work, and it is a lot of work, but if you keep at it then it will pay off!

- **Pre-class assignments** (20% of final grade) (3 per week)
  - **What?** Small amount of reading, and two or three basic exercises covering the assigned reading and/or the content in the previous class. Graded for completion only.
  - **Why?** To prepare you for lectures, to help you keep up with the course material, and to allow me to identify things to emphasise or cover in more detail in class.
- **Problem sheets** (30% of final grade) (1 per week, except first week)
  - **What?** Several challenging questions covering material covered in class. Graded for mathematical correctness and proof-writing quality. Can be re-submitted to achieve a better grade.
  - **Why?** To give you the opportunity to show off what you've learnt, and to allow me to give you individual feedback on your progress.
- **Classwork** (20% of final grade) (all classes)
  - **What?** In-class activities (individually or in small groups) and quizzes.
  - **Why?** To keep you engaged with the material as it is presented, to put into practice the social aspect of mathematics, and to prevent you from falling asleep.
- **Final project** (30% of final grade) (due morning after last day of class)
  - **What?** Extended write-up about a mathematical problem or theorem. A detailed description of the final project can be found in Appendix E of the course notes.
  - **Why?** To bring together all the skills you have learnt in the course, and to expose you to the kind of mathematics done by mathematicians.

In order to pass the course, you must attain a final grade of 60% or more, and you must make a serious attempt at all questions on all pre-class assignments and problem sheet questions. The grade boundaries for those who pass are:

A: 90–100% **B:** 80–89% **C:** 70–79% **D:** 60–69%

Fractional final percentages will be rounded up. If you get 100% then you get a victory t-shirt.

3 Schedule

	Date		Topic	PCA	PS
Week 1	Monday	29th June	Introduction		
	Tuesday	30th June	Propositions and quantifiers	✓	
	Wednesday	1st July	**Proofs and problems**	✓	
	Thursday	2nd July	Sets	✓	
	Friday	3rd July	*(No class)*		
Week 2	Monday	6th July	Induction and recursion	✓	
	Tuesday	7th July	More induction and recursion		✓
	Wednesday	8th July	**Proofs and problems**	✓	
	Thursday	9th July	Counting principles	✓	
	Friday	10th July	More counting principles		
Week 3	Monday	13th July	Binomials and factorials	✓	
	Tuesday	14th July	More binomials and factorials		✓
⋆	Wednesday	15th July	**Proofs and problems**	✓	
	Thursday	16th July	Division	✓	
	Friday	17th July	More division		
Week 4	Monday	20th July	Primes and irreducibles	✓	
	Tuesday	21st July	More primes and irreducibles		✓
	Wednesday	22nd July	**Proofs and problems**	✓	
	Thursday	23rd July	Modular arithmetic	✓	
	Friday	24th July	More modular arithmetic		
Week 5	Monday	27th July	Relations	✓	
	Tuesday	28th July	More relations		✓
	Wednesday	29th July	**Proofs and problems**	✓	
	Thursday	30th July	Functions	✓	
	Friday	31st July	More functions		
Week 6	Monday	3rd August	Cardinality	✓	
	Tuesday	4th August	More cardinality		✓
	Wednesday	5th August	**Proofs and problems**	✓	
	Thursday	6th August	Sets with structure	✓	
	Friday	7th August	More sets with structure		
	Saturday	8th August	**Final project due at 9am**	(18)	(5)
	Tuesday	11th August	**Final grades on SIO**		

PCA: pre-class assignment due (18 total); **PS**: problem sheet due (5 total); ⋆ My birthday, bring cake.

4 Policies

It is not in my interests to make your life difficult, but since some instructors are different from others, it is important that I clarify my stance on various issues. As such, please read this section at least once, and refer back to it if any of the issues apply to you at any point during the course!

Academic honesty and integrity

My stance on academic honesty is fairly simple: **all work you submit should be your own work and should reflect your understanding.** If you're ever in need of additional help and are in any doubt about whether you're going to break any rules accidentally, come and see me for help.

- **Collaboration.** Speak to each other! Learning collaboratively makes you think in ways you never thought to think before, which is great. But when it comes to discussing homework or the final project, any records you make during the discussion must be destroyed before you make any kind of write-up. This includes office hours! All homework you submit, including the final project, should be written individually and independently by you. If you have collaborated with anyone then you should declare who you worked with and the nature of your discussion (e.g. 'I discussed Q4 with Carl Gauss, who said I should use induction').

  You will **not** be penalised for declared homework collaboration! However, undeclared collaboration is plagiarism, and collaboration outside of the above guidelines is considered cheating.

  For example, if you write anything on a whiteboard and your friend writes things down on paper, you and your friend should erase the whiteboard and throw away the paper before writing anything permanent, such as written-up solutions or rough work that you keep.

- **External resources.** You are encouraged to use external resources, such as textbooks and the internet, as long as it is in such a way that you enhance your understanding and assessed material isn't trivialised by your use of such resources. If you use an external resource then you should cite your source: book title and author, web page URL, etc.

  You will **not** be penalised for declared use of external resources! However, undeclared use of external resources is plagiarism, and use of external resources outside of the above guidelines is considered cheating.

  For example, if you're stuck on a problem, find a solution on the internet and then copy or paraphrase it in your answer, this *is not permitted* and constitutes cheating. However, if you're stuck on a problem, read up on the underlying theory and then work out a solution by yourself, this *is permitted* so long as you cite your source.

Homework submission

Please do the homework and submit it on time—it's for your benefit, not mine... trust me, grading is a nightmare! It is your responsibility to ensure that I receive homework when it's due. If there are any special circumstances, such as illness or planned absence, you should let me know as soon as possible (and certainly before the deadline). The sooner you ask for an extension or exemption, the more likely I am to grant it. If I find an unannounced homework in my mailbox or under my office door I simply won't be able to accept it for credit. Late homework will be accepted for the purposes of passing the course, but will not gain credit.

Attendance

Classes will be awesome and fun so you'll probably want to attend, and it's in your interests to do so anyway since 20% of your grade comes from in-class activities. Some of these activities might be at the beginning of class, too, so please be on time. (I won't wait for you!)

If you can't come because you're sick or there's some kind of unforeseen emergency then send me an email as soon as possible: I will send you notes for the class and some extra exercises to make sure you're up to speed for class the next day. If you're absent more than once then I will need some form of official documentation, otherwise I won't be able to give you any make-up credit.

Accommodations

Please ensure I'm aware of any accommodations that need to be made, such as extra time on in-class assessments or large text, and give supporting documentation from the Office of Disability Resources.

Talk to me

I want you to learn lots and I want you to enjoy taking this course. So that I can find out if this is happening, I encourage feedback—be it positive or negative—on all aspects of the course at any time during the semester. For example, if something I'm doing is making it difficult for you to learn, then say something before it's too late; or if you particularly enjoyed something we did in class, say so so that we can do it again. You can do this by just speaking to me, by sending me an email, or by using the anonymous comments form, details of which will be circulated in class. Please bear in mind that I cannot reply individually to anonymous feedback. Giving feedback will in no way affect your grade, positively or negatively.